# RELATIVISTIC EFFECTS ON THE MOTION OF ASTEROIDS AND COMETS

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We study the effects arising from relativistic perturbations 011 the motion of asteroids and comets and show that for a number of such objects, inclusion of relativistic contributions in the equations of motion gives rise to significant improvements in the orbital solutions. Furthermore we argue that ignoring relativistic corrections to the equations of motion, while using masses derived from relativistic ephemerides yields incorrect solutions corresponding to an inconsistent, non-Newtonian, non-relativistic 1110(1 ('1.

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## 1. INTRODUCTION

Relativistic effects, arising from the theory of general relativity (Einstein 1915) are important in modeling the dynamics of comets and asteroids in the inner solar system. In attempts to verify the predictions of general relativity, Shapiro et al. (1968), Lieske and Null (1969), and Shapiro (1971) found that the non-Newtonian motion for asteroid 1566 Icarus was consistent with the predictions of general relativity but orbital solutions for Icarus were of marginal use for determining the solar quadrupole moment or the mass of Mercury. Using observations over the interval 1949 through 1968, these authors pointed out that orbital solutions for Icarus using relativistic equations of motion provided significantly improved results. Sitarski (1992) reached the same conclusion when he updated the orbit of Icarus using the optical observations from 1949 through 1987.

The generic first order relativistic effects are of order $v^2/c^2$ ($GM_s/rc^2$), where $c$ is the speed of light, $v$ and $r$ are repectively the speed of the object and its distance from the Sun, $G$ is the gravitational constant and $M_s$ is the solar mass. Thus, the fractional first order relativistic effects at about 1 AU are of the order of $1 \times 10^{-8}$. The current positional measurement accuracy for asteroids and comets is of the order of a few tenths of a second of arc. For one orbital period of asteroid Icarus, the perihelion precession is about the same. For some asteroids and comets with eccentric orbits, small semi-major axes, and long observational data intervals, relativistic effects will be necessary to properly fit the observations.

We have used all the available optical and radar data to improve the orbits of six asteroids, including Icarus, whose motions are significantly modified by the effects of general relativity. In addition, we will show that even when the orbital rms residuals are not improved when relativistic equations of motion are substituted for the commonly used Newtonian equations, significant orbital errors for many asteoids and comets will be avoided when relativistic equations of motion are employed in conjunction with modern planetary ephemerides. Since relativistic corrections are often incorrectly assumed negligible in the motions of asteroids and comets, we will begin with an overview of the various relativistic effects that come into play during the orbit determination process.

## II. RELATIVISTIC EFFECTS IN MEASUREMENT MODELS

Measurements of the position of an asteroid or comet are currently performed using either optical or radar telescopes. In such measurements, relativistic effects are most prominent only when the line-of-sight grazes the Sun. Although optical observations are not possible near the Sun, radar observation models may have to be modified by the relativistic time delay due to the propagation of the radar signal in the gravitational potential of the Sun (Shapiro 1964, Standish 1990). However, for the processing of current asteroid and comet radar observations, these corrections are usually much smaller than the errors in the delay measurements themselves (Yeomans *et. al.* 1992).

## III. RELATIVISTIC EFFECTS IN THE EQUATIONS OF MOTION

There are a number of relativistic effects which contribute to the motion of a comet or asteroid. Here we consider a simple model of an asteroid or comet falling freely in the gravitational field of the Sun. We ignore all effects arising from the planetary perturbations. These can be included in a straightforward manner in a more general scheme but at the moment we note that the largest relativistic effects arise from the presence of the Sun. The metric tensor for a single spherically-symmetric gravitational source can be written as the slow-motion, weak-field approximation of the Schwarzschild (1916) metric:

$$g_{00} = -1 + 2\frac{\mu}{r} - 2\frac{\mu^2}{r^2}, \tag{3.1}$$

$$g_{0i} = 0, \tag{3.2}$$

$$g_{ij} = \delta_{ij}(1 + 2\frac{\mu}{r}). \tag{3.3}$$

Here $\mu$ is the Schwarzschild radius of the Sun given in terms of the post-Newtonian mass of the Sun $\mu = GM_{PN}/c^2 = GM_s/c^2 \approx 1.5$ km and r is the object's distance from the Sun. The first two terms in $g_{00}$ and the first term in $g_{ij}$ give rise to the Newtonian gravitational force. The last term in $g_{00}$ provides 1/3 of the relativistic precession of the orbital line of apsides, or perihelion. The $g_{0i}$ components of the metric represent contributions arising from the motion of the source. In the case of a single object freely falling in the field of the Sun, the Sun's velocity is negligible. If one were to include the planets as gravitational sources, as in the JPL ephemeris development models, these terms would have non-negligible contributions. The last term in $g_{ij}$ represents the curvature of space. This term provides 2/3 of the perihelion precession effect and other curvature effects such as the 19 milliarcsec/year geodetic precession of the earth's inertial frame (Lieske *et. al.* 1977; Shahid-Saless and Ashby 1988).

The equations of motion for an object freely falling in the gravitational field described by the above metric can be derived from the least action principle. The invariant interval in four dimensions is given by:

$$ds^2 = -g_{\mu\nu}dx^{\mu}dx^{\nu} \tag{3.4}$$

$$(1 - 2\frac{\mu}{r} + 2\frac{\mu^2}{r^2})(dx^0)^2 - (1 + 2\frac{\mu}{r})\delta_{ij}dx^i dx^j, \tag{3.5}$$

where $\mu$ and $\nu$ are the space-time indices running from 0 to 3, $i$ and $j$ are the spatial coordinate indices running from 1 to 3 and the Einstein summation convention is used. The classical action $A$ is the integral of $ds$ along the path of the particle.

$$A = -mc^2 \int_{path} ds \tag{3.6}$$

$$= -mc^2 \int_{path} \left(\frac{ds}{dx^0}\right) dx^0, \tag{3.7}$$

where $m$ is the mass of the particle in motion. From classical Lagrangian formalism we know that the action is the integral of the Lagrangian $L$ over time, along the path of the particle:

$$A = \int_{path} L dx^0 \tag{3.8}$$

Thus comparing Eq. (2.8) with Eq. (2.7) and using Eq. (2.5), the Lagrangian is given by:

$$L = -mc^2[(1 - 2\frac{\mu}{r} + 2\frac{\mu^2}{r^2}) - (1 + 2\frac{\mu}{r})\,\delta_{ij}v^i v^j]^{\frac{1}{2}}, \tag{3.9}$$

where $v^i \equiv dx^i/dx^0$. The equations of motion can now be derived using the Euler-Lagrange equations:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial v^i}\right) - \frac{\partial L}{\partial x^i} = 0. \tag{3.10}$$

We get:

$$\frac{d^2\mathbf{r}}{c^2dt^2} = -\frac{\mu}{r^3}\mathbf{r} + \frac{\mu}{r^3}\left[\left(4\frac{\mu}{r} - \frac{v^2}{c^2}\right)\mathbf{r} + 4\frac{(\mathbf{r}\cdot\mathbf{v})\,\mathbf{v}}{c^2}\right], \tag{3.11}$$

which agrees with the single-source relativistic equations of motion used in modeling solar system dynamics (Anderson *et. al.* 1975; Moyer 1971). We can see that the first term on the right hand side of the above equation is the Newtonian gravitational acceleration. The remaining terms are the relativistic corrections which, in part, give rise to the relativistic precession of the perihelion. The relativistic advance of the perihelion can best be derived using the Hamiltonian formulation of the equations of motion. The canonical momenta are given, to the desired order of accuracy, by:

$$\pi^i = \frac{\partial L}{\partial v^i} \tag{3.12}$$

$$= v^i\left[1 + \frac{1}{2}v^2 + 3\frac{\mu}{r}\right]. \tag{3.13}$$

The Hamiltonian is now given by

$$H = \pi \cdot \mathbf{v} - L \tag{3.14}$$

$$= \frac{1}{2}\pi^2 - \frac{\mu}{r} + \left(\frac{\mu}{r}\right)^2 - \frac{1}{8}\left[\pi^2 + 2\frac{\mu}{r}\right]^2 - \frac{\mu}{r}\pi^2. \tag{3.15}$$

The first two terms in the total Hamiltonian correspond to the Newtonian gravitational force. We can now treat the remaining terms in the Hamiltonian as a perturbation $H_1$. This perturbation depends only on three orbital elements; the mean anomaly $M$, the semimajor axis $a$ and the eccentricity $e$. One can rewrite $H_1$ as:

$$H_1 = -\frac{\mu^2}{2a^2} + 2\frac{\mu^2}{ra} - 3\frac{\mu^2}{r^2}, \tag{3.16}$$

where we have used the energy conservation equation:

$$v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right). \tag{3.17}$$

The advance of the argument of perihelion is now given by (Bertotti and Farinella 1990):

$$\frac{d\omega}{dt} = \frac{(1-e^2)^{\frac{1}{2}}}{na^2 e}\frac{\partial H_1}{\partial e}, \tag{3.18}$$

noting that $r = a(1 - e\cos E)$, $E$ being the eccentric anomaly and $n$ being the mean motion $2\pi/T$. The first two terms in Eq. (2.16) do not give rise to a secular advance in the argument of perihelion. The contribution of the last term can be calculated by averaging over one period and noting that

$$\int_0^{2\pi} \frac{dE}{1 - e\cos E} = \frac{2\pi}{\sqrt{1-e^2}}. \tag{3.19}$$

We get:

$$\Delta\omega = \frac{6\pi\mu}{a(1-e^2)}, \tag{3.20}$$

or $\Delta\omega = 0.0384/a(1-e^2)$ arc seconds per revolution when a is in astronomical units. Since most observations of asteroids and comets at this time are optical angular positions, the perihelion precession is perhaps the most measurable effect. This effect is most noticable for eccentric orbits with small semi-major axes. One can compute the magnitude of the perihelion advance for all currently known asteroids and comets to see whether relativistic effects should be included in modeling the dynamics of such objects. For the 15 objects most affected by relativistic perturbations, Table 1 gives the name and number of the asteroid followed by its amount of orbital precession (perihelion) per year. The next three columns give the number of years for which observations exist, the orbital period and the total relativistic precession to mid-1993.

Assuming that the current accuracy in determining the angular position of an asteroid is a few tenths of a second of arc, the relativistic contribution to precession of the perihelion should be clearly detectable for the asteroids near the top of the list.

## IV. RELATIVISTIC V.S. NEWTONIAN DYNAMICS

The Schwarzschild metric is the vacuum solution to the spherically symmetric Einstein's field equations, relevant only when there exists a single gravitational source. For the motion of solar system bodies, the appropriate metric to use is the many-body post-Newtonian metric such as the one devised by Will and Nordtvedt (1972). This metric is the weak-field, slow-motion solution to Einstein's field equations for an arbitrary number of sources and thus a better representation of the solar system. The JPL planetary ephemeris development program utilizes the point-mass version of the post-Newtonian metric to include relevant relativistic effects in the solar system. The solar system parameters derived by this development effort are therefore accurate to post-Newtonian order. In comparing the observed positions of comets and asteroids with those predicted from theoretical models, one must also realize that the mass of the Sun determined by the solar system ephemeris effort includes not only relativistic corrections arising from the post-Newtonian formulation of the equations of motion, but also includes corrections devised by the IAU to keep the relativistic equations of motion in the barycentric frame simple. Corrections arising from the IAU choice of coordinates need be utilized in the definition of space-time coordinates which we will consider shortly. Corrections to the Newtonian equations of motion arising from post-Newtonian formulation are of the form of Eq. (?.11). Discrepencies arising from exclusion of the relativistic corrections can be understood by comparing Eq. (2.11) with Newton's second law:

$$\frac{d^2\mathbf{r}}{dt^2} = -\frac{GM_N}{r^3}\mathbf{r}, \tag{4.1}$$

where $M_N$ would be the mass of the Sun derived from Newtonian ephemerides. Comparison with Eq. (2.11) shows us that if one were to use the post-Newtonian mass, used in Eq. (2.11) in Newton's second law, a fit to data could still be made because the leading terms have the same functional $1/r^2$ dependence on r. However, the neglected terms in Eq. (2.11) would still contribute on average by a significant amount. Thus the fit would yield a semi-major axis that is incorrect by an amount:

$$\delta a = a_r - a_{n.r} \approx \frac{1}{2}[4\frac{\mu}{r} - \langle\frac{v^2}{c^2}\rangle]a_r \approx 10^{-8}a_r \tag{4.2}$$

This would be in addition to whatever error came about by not having modeled the relativistic precession of perihelion. Of course one remedy for correcting the semi-major axis to post-Newtonian order is to rescale the solar mass. In this way the Newtonian equations of motion would, on the average, agree with the relativistic version except for the fact that the relativistic precession of perihelion would still not be modeled. Hence, the orbital determination of objects using Newtonian equations of motion but utilizing the solar mass derived from JPL planetary ephemerides (i. e. $M_{TDB}$) will in general have inconsistencies. Thus although the motions of many asteroids and comets may not be sensitive to true relativistic effects, their dynamical modeling, if not relativistically formulated, is in error.

As mentioned earlier, corrections arising from the choice of using TDB coordinates are also important in correct dynamic modeling of solar system objects. Relativistic equations of motion are usually written in terms of the Post-Newtonian coordinates, defined by the post-Newtonian formalism developed by Will and Nordtvedt (1972). This choice of coordinates is only a matter of convention since Einstein's field equations are generally covariant and thus physical measurements are independent of the choice of coordinates. Nevertheless according to the IAU convention, the barycentric time. ($t_{TDB}$) is to be related to an ideal Earth-borne clock time ($t_E$) by a periodic coefficient, namely (Hellings 1986)

$$t_{TDB} = (1 + U + \frac{1}{2}\frac{v^2}{c^2} - \langle U + \frac{1}{2}\frac{v^2}{c^2}\rangle)t_E. \tag{4.3}$$

Here $U$ is the total gravitational potential on the Earth's surface. The Post-Newtonian time coordinate, which is used in the modeling of the motion of the planets, is related to TDB by:

$$t_{TDB} = (1 - U - \frac{1}{2}\frac{v^2}{c^2})t_{PN}. \tag{4.4}$$

In order to keep the equations of light propagation unchanged (i.e. keeping the speed of light constant) one has to also re-scale the spatial coordinates such that

$$dx_{TDB} = (1 - U - \frac{1}{2}\frac{v^2}{c^2})dx_{PN} \equiv (1 - L)dx_{PN}. \tag{4.5}$$

However this will enter the equations of motion of ordinary matter by re-scaling the components of the metric tensor:

$$g^{\mu\nu}_{TDB} = (1 - L)^2 g^{\mu\nu}_{PN}. \tag{4.6}$$

In turn, this will affect the post-Newtonian equations of motion unless one rescales all the masses. If one tries to keep the form of the post-Newtonian equations unchanged, the TDB masses have to be M-scald with respect to PN masses by:

$$M_{TDB} = (1 - L)M_{PN}. \tag{4.7}$$

The magnitude of $L$ is 1 .55052 X10- [8]. Integration of the equations of motion in the relativistic formalism is self-consistent and has been shown to be so previously (Martin *et. al.* 1985; Ries *et. al.* 1988). The masses of the Sun and the planets derived from the J]'], planetary ephemerides (DE 200, de.) include relativistic corrections.

Thus, using the Newtonian equations of motion along with the relativistically determined masses would in general yield a solution which is neither relativistically correct (no relativistic precession) nor non-relativistically consistent- a hybrid solution.

## V. ORBITA , SOLUTION EXPERIMENTS

We performed experiments to fit the relativistic and the hybrid models described above to the observational data of a subset of objects for which the cumulative relativistic precession of the perihelion is large. The subset was chosen from some 156 comets and asteroids commonly under study. We studied the full set by looking at the rate of perihelion precession, the observational period, the number of actual observations and the total amount of precession during the period of observation. Table 1 lists the top 15 such objects ordered in decreasing total perihelion precession. The asteroid 1566 Icarus is at the top of the list. We have performed orbital fits to the data available for several asteroids listed in Table I. For six objects, Table II includes the number and type of the available observations, the observation interval and the RMS residuals for the relativistic and the hybrid models. The ordering here is by the amount of improvement in the RMS residual which for Icarus is as much as 30%. It is evident that the dative improvement to the orbital solutions, when a consistent relativistic formulation is employed instead of the hybrid model, is inversely correlated to the orbital semi-latus rectum ($p = a(1 - e^2)$) and proportional to the observation interval and the amount of data available. Improved orbital elements for the six asteroids in table II are given in table III.

## TABLES

TABLE 1. 15 asteroids with the largest perihelion precession rates.fl

| Object | Prec. Rate. (arc. sec. / yr) | obs. Interval (yrs.) | period (yrs.) | Tot. Prec. to 5/93 (arc. sec.) |
|---|---|---|---|---|
| 1566 Icarus | 0.101 | 43 | 1.119 | 4.34 |
| 2062 Ate]\ | 0.043 | 3'7 | 0.950 | 1.59 |
| 1862 Apollo | 0.021 | 57 | 1.785 | 1.20 |
| 2100 Ra-Shalom | 0.075 | 15 | 0.759 | 1.13 |
| 1685 Toro | 0.022 | 44 | 1.598 | 0.97 |
| 2101 Adonis | 0.019 | 48 | 2.567 | 0.91 |
| 3'753 (I 986 'J'()) | 0.053 | ]9 | 0.997 | 0.90 |
| 433 Eros | 0.016 | 56 | 1.761 | 0.90 |
| 1620 Geographos | 0.025 | 35 | 1.389 | 0.88 |
| 3200 Phaethon | 0.101 | 8 | 1.423 | 0.81 |
| 1865 Cerberus | 0.040 | 18 | 1.123 | 0.72 |
| 5143 Heracles | 0.021 | 30 | ?.484 | 0.63 |
| 1951 Lick | 0.017 | 36 | 1.640 | 0.61 |
| 1627 Ivar | 0.010 | 61 | 2.544 | 0.61 |
| 2340 Hathor | 0.074 | 7 | 0.775 | 0.52 |

[a] Orbital solutions for 6 asteroids that are affected by relativistic effects. In each case, JPL development ephemeris DE200 was used. For each object listed, the number of optical and radar (time delay and Doppler) observations are given along with the data interval over which the orbit was computed. For the RMS orbit residual, the first line for each object gives the value when purely Newtonian equations of motion were employed while the second line gives the same information when relativistic equations of motion were used.

TABLE 11. The six asteroids under study, their observations and their RMS residuals. For those asteroids without radar data, the units of the RMS residuals are in arc seconds. Otherwise the RMS residuals are normalized, unitless values. That is, each orbit residual (in arcsec, Hz, or microseconds) was divided by its observation weight in the same units before the RMS value was computed.

| | Observations | | | | RMS |
|---|---|---|---|---|---|
| | optical | delay | Doppler | Data Interval | Resid. |
| 1566 Icarus | 466 | - | 9 | 06/27/49 - 10/12/92 | 1.42 |
| | | | | | 0.99 |
| 1862 Apollo | 99 | 4 | 8 | 01/27/32- 12/29/80 | 1.33 |
| | | | | | 1.17 |
| 3200 Phaethon | 92 | | | 10/1?/83- 11/29/92 | 1.01 |
| | | | | | 0.97 |
| 3753 (1986 TO) | 67 | | | 10/17/73- 08/25/92 | 1.08 |
| | | | | | 1.06 |
| 2100 Ra-Shalom | 75 | - | 2 | 10/0?/70- 10/09/91 | 0.99 |
| | | | | | 0.98 |
| 2062 Aten | 65 | - | - | 12/17/55- 10/06/92” | 1.21 |

1.21

TABLE 111, Orbital elements for 6 asteroids whose motion is significantly affected by general relativistic effects. Each orbit was computed using J 1'1 ,'s 1 )evelopment Ephemeris DE200 with the Earth and moon perturbations treated seperately. The normalized rm s orbital residuals and the employed astrometric data are given in table 11. The angles are given in degrees and refer to the ecliptic plane and J2000 equinox. F 'or the six orbital elements corrected in the orbit determination p rocess, the formal standard deviations are given in parentheses in units of the last decimal place. The given orbital elements are respectively the eccentricity, the perihelion distance in AU, the time of perihelion passage ('1'1)11), the argument of perihelion, the longitude of the ascending node, the inclination, semi-major axis in AU and the mean anomaly in degrees.

| | 1566 Icarus | 1862 Apollo | 3200 Phaethon |
|---|---|---|---|
| Epoch | 1994 Feb 17. 0 (TDB) | 1994 Feb 17.0 (T1 )B) | 1994 Feb 17.0 (T1 )B) |
| e | ().826694124 (109) | 0.559941362 (33) | 0.890151589(136) |
| q | 0.186836585 (1 18) | 0.647353126 (49) | 0.139653993(176) |
| T' | 1994 Jan 27.7429529 (217) | 1995 Jan 2.7165230(102) | 1993 Sep 12.7348078 (2513) |
| w | 31.2248612 (127) | 285.6391790(584) | 32].8104926 (355) |
| Node | 88.1537825(55) | 35.9330697(579) | ?65.5970894 (370) |
| i | '22.8790200(249) | 6.3562961 (239) | 22.0974651 (363) |
| a | 1.078074153 | 1.471061059 | 1.271333759 |
| MA | 17.8364024 | 1s3.386s217 | 108,1302769 |
| | 3753 (1986 TO) | 2100 Ra-Shalom | 2062 Aten |
| Epoch | 1994Feb 1'7.0('1'1 )11) | 1994 Feb 17.() ('I'1 )11) | 1994Feb 17.() ('I'1 )11) |
| e | 0.514811131 (2438) | 0.43645 6323(241) | 0.182583091 (379) |
| q | ().484()91097 (2440) | 0.468895485 (198) | 0.790133591 (363) |

| | | | |
|---|---|---|---|
| T' | 1994 Jan 15.4? 33736 (1890) | 1994 Mar 31.4564282 (705) | 1994Jun 8.6636837 (1 129) |
| w | 43.6374549 (3497) | 355.944?840 (504) | 147.9154590 (1198) |
| Node | 126.3950772(4619) | 170.9613315 (273) | 108.6855?38 (236) |
| i | 19.8110943(1086) | 15.7555290(251) | 18.9319604 (633) |
| a | *0.997737433* | *0.832048169* | 0.966622518 |
| MA | 32.2170508 | 304.865324'2 | ?44,1941062 |

## VI. CONCLUSIONS

We have studied the prominent effects arising from the inclusion of general relativity in modeling the dynamics of asteroids and comets. We have demonstrated that for a particular set of objects with small semi-major axes and large eccentricities, relativistic effects can play a major role in dynamical modeling of such objects. An important purpose of this work has been to show that for such objects, exclusion of relativity effects, while using solar-system masses derived from relativistically derived ephemerides (such as DE200), will in general result in inconsistent solutions for the dynamical parameters. We have shown that such orbital solutions are neither relativistically correct nor consistent with a completely non-relativistic model; i. e. they correspond to a hybrid model. Finally we performed experiments to derive the RMS residual resulting from solutions to the available data using the current JPL programs. This was done with both the fully relativistic model and the hybrid version. The conclusion is that in the more extreme relativistic cases, the relativistic model yields smaller residuals. In the case of the asteroid 1566 Icarus, the improvement in the RMS residual is as large as 30%.

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